

sec 4

sheet 3

PAGE

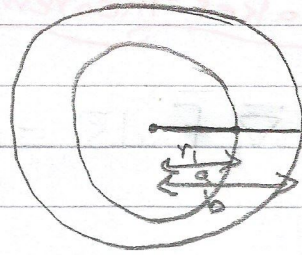
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⑨

$$\vec{F} = 5r^3 \vec{r}$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b 5r^3 r dr$$

$$= \int_a^b 5r^4 dr = r^5 \Big|_a^b = b^5 - a^5$$



sheet 3 ans

① $\frac{1}{3}$

② $\frac{1}{2}$

③ 1

④ 96π

⑤ 236

⑥ $-\frac{8}{3}$

$-\frac{44}{3}$

2π

⑦ $\phi = 3x^2y^2 - xy^2$

⑧ 5

⑨ $b^5 - a^5$

sheet 4

* divergence Theorem :-

$$\oint_S \vec{F} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{F} dv$$

* Stokes Law :-

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

$$\textcircled{3} \quad \vec{A} = a x \vec{i} + b y \vec{j} + c z \vec{k}$$

$$\oiint \vec{A} \cdot d\vec{s} = (a + b + c) V$$

According to ~~div~~ divergence Theorem

$$\begin{aligned} \text{L.H.S} &= \oiint_S \vec{A} \cdot d\vec{s} = \iiint_V (\vec{\nabla} \cdot \vec{A}) dV \\ &= \iiint_V (a + b + c) dV = (a + b + c) \iiint_V dV \\ &= (a + b + c) V = \text{R.H.S} \end{aligned}$$

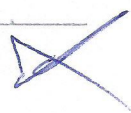
$$\vec{\nabla} \cdot \vec{A} = a + b + c$$

$$\textcircled{5} \quad \iiint_V \frac{dV}{r^2} = \oiint_S \frac{\vec{r} \cdot \vec{n}}{r^2} d\vec{s} = d\vec{s}$$

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r^2} \quad \vec{F} = \frac{\vec{r}}{r^2}$$

$$\begin{aligned} \vec{\nabla} \cdot \vec{F} &= \vec{\nabla} \cdot \left(\frac{\vec{r}}{r^2} \right) = \frac{1}{r^2} (\vec{\nabla} \cdot \vec{r}) + \vec{r} \cdot \left(\vec{\nabla} \frac{1}{r^2} \right) \\ &= \frac{3}{r^2} + \vec{r} \cdot \left(\frac{-2}{r^3} \right) \textcircled{A} \rightarrow \left| \frac{\vec{r}}{r} \right| \\ &= \frac{3}{r^2} - \frac{2}{r^2} = \frac{1}{r^2} \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= \oiint_S \frac{\vec{r}}{r^2} d\vec{s} = \oiint_S \vec{F} \cdot d\vec{s} \\ &= \iiint_V \vec{\nabla} \cdot \vec{F} dV = \iiint_V \frac{1}{r^2} dV = \text{L.H.S} \end{aligned}$$



7) $\oint \frac{\partial \psi}{\partial x} dy - \frac{\partial \psi}{\partial y} dx = 0$ where C is a closed curve in the region R ,

prove that is $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

$$F_1 = \frac{\partial \psi}{\partial x}$$

$$F_3 = 0$$

$$F_2 = -\frac{\partial \psi}{\partial y}$$

$$\begin{aligned} \nabla \times \vec{F} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \psi}{\partial x} & -\frac{\partial \psi}{\partial y} & 0 \end{vmatrix} \\ &= \left(\frac{\partial}{\partial z} \left(-\frac{\partial \psi}{\partial y} \right) - 0 \right) \hat{i} - \left(\frac{\partial}{\partial z} \left(\frac{\partial \psi}{\partial x} \right) - 0 \right) \hat{j} \\ &= \frac{\partial^2 \psi}{\partial x^2} \hat{j} + \frac{\partial^2 \psi}{\partial y^2} \hat{j} = 0 \end{aligned}$$

8) Evaluate $\oint (y^2 z \hat{i} + x^3 \hat{j} + xz \hat{k}) \cdot d\vec{s}$

$-1 \leq x \leq 1, -1 \leq y \leq 1$ and $0 \leq z \leq 2$

Sol.

$$\oint \vec{F} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{F} dV$$

$$\nabla \cdot \vec{F} = 0 + 0 + x = x$$

$$\int_0^2 \int_{-1}^1 \int_{-1}^1 x dx dy dz = \int_0^2 \int_{-1}^1 \frac{x^2}{2} dy dz$$

$$= 0$$